INTERNATIONAL BACCALAUREATE

Mathematics: applications and interpretation

MAI

EXERCISES [MAI 3.6-3.7]

TRIGONOMETRIC CIRCLE – RADIANS –TRIANGLES II

Compiled by Christos Nikolaidis

A. Paper 1 questions (SHORT)

TRIGONOMETRIC CIRCLE AND RADIANS

1. [Maximum mark: 6]

(a) Express the following angles in radians; give your answer in terms of π.

(i) 20° (ii) 18° (iii) 540° [3]

(b) Express the following angles in degrees

(i) $\frac{\pi}{18}$ rad (ii) $\frac{\pi}{5}$ rad (iii) 2.5π . [3]

....

....

....

....

....

....

....

....

....

....

....

....

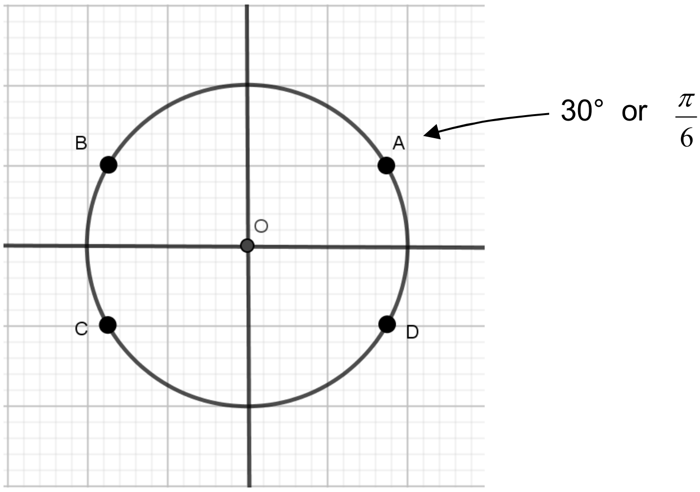
....

....

....

2. [Maximum mark: 8]

In the diagram below, the point A represents the angle of 30°, or otherwise $\frac{\pi}{6}$ rad, on the unit circle.



(a) Write down the values corresponding to the points B, C and D

(i) in degrees in the interval $0° \leq \theta < 360°$

(ii) in radians in the interval $0 \leq \theta < 2\pi$ [4]

	A	B	C	D
in degrees	30°			
in radians	$\frac{\pi}{6}$			

(b) Suppose now that C represents the angle of 220°, or otherwise $\frac{11\pi}{9}$ rad,

Complete in a similar way as in (a) the following table: [4]

	A	B	C	D
in degrees			220°	
in radians			$\frac{11\pi}{9}$	

....

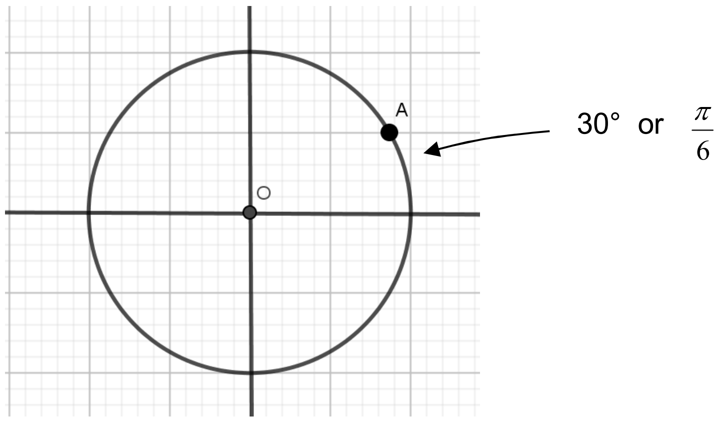
....

....

....

3. [Maximum mark: 8]

In the diagram below, the point A represents the angle of 30°, or otherwise $\frac{\pi}{6}$ rad, on the unit circle.



The general formula for the angles corresponding to point A is

in **degrees**: $30° + 360°k \qquad k \in Z$

in **radians**: $\frac{\pi}{6} + 2k\pi \qquad k \in Z$

Determine the values of the angle at point A the following intervals:

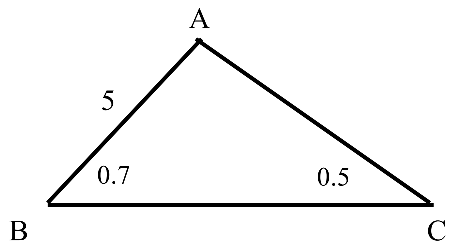
	In degrees	
2nd period backwards	$-720° \le \theta < -360°$	
1st period backwards	$-360° \le \theta < 0°$	
1st period	$0° \le \theta < 360°$	30°
2nd period	$360° \le \theta < 720°$	
3rd period	$720° \le \theta < 1080°$	

in radians	
$-4\pi \le \theta < 2\pi$	
$-2\pi \le \theta < 0$	
$0 \le \theta < 2\pi$	$\frac{\pi}{6}$
$2\pi \le \theta < 4\pi$	
$4\pi \le \theta < 6\pi$	

[8]

4. [Maximum mark: 8]

The angles of the triangle ABC below are given in radians.



(a) Find the value of angle A (in radians), correct to 2 dp. [2]

(b) Find AC. [2]

(c) Find BC. [2]

(d) Find the area of the triangle. [2]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

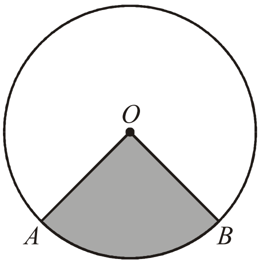
....

....

ARCS AND SECTORS (IN RADIANS)

5. [Maximum mark: 8]

O is the centre of the circle which has a radius of 10 cm. The size of $A\hat{O}B$ is 1.5 rad.



(a) Find the **lengths** of the minor arc AB and of the major arc AB. [3]

(b) Find the **areas** of the minor sector (shaded region) and of the major sector. [3]

(c) Find the **perimeters** of the minor sector (shaded region) and of the major sector. [2]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

6. [Maximum mark: 6]

The following diagram shows a circle of centre O, and radius 15 cm. The arc ACB subtends an angle of 2 radians at the centre O.

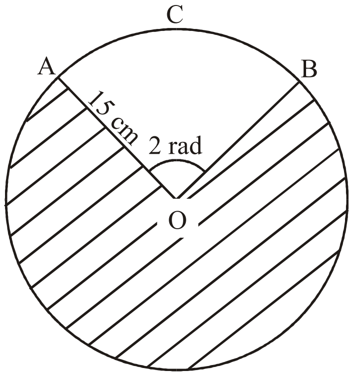


Diagram not to scale

$A\hat{O}B = 2$ radians
OA = 15 cm

(a) Find the length of the arc ACB; [2]

(b) Find the area of the shaded region. [4]

....

....

....

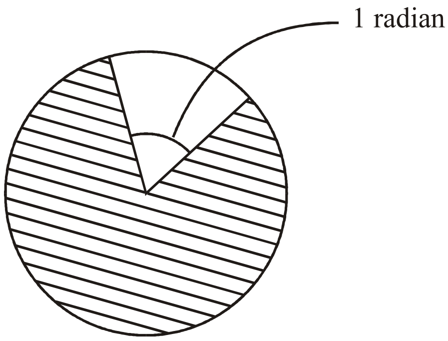
....

....

....

7. [Maximum mark: 4]

The diagram shows a circle of radius 5 cm. Find the perimeter of the shaded region.



....

....

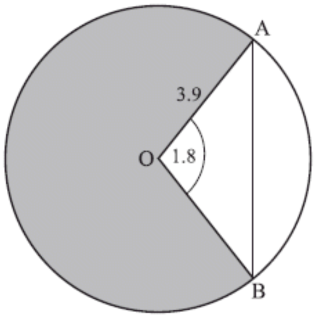
....

....

....

8. [Maximum mark: 7]

The circle shown has centre O and radius 3.9 cm. Points A and B lie on the circle and angle AOB is 1.8 radians.



(a) Find AB. [3]

(b) Find the area of the shaded region. [4]

....

....

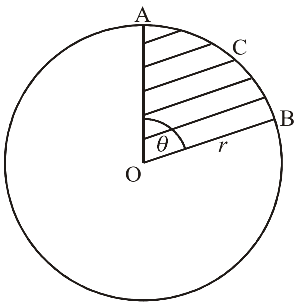
....

....

....

9. [Maximum mark: 6]

The following diagram shows a circle of centre O, and radius *r*. The shaded sector OACB has an area of $27\ \text{cm}^2$. Angle $\hat{AOB} = \theta = 1.5$ radians.



(a) Find the radius. [4]

(b) Calculate the length of the minor arc ACB. [2]

....

....

....

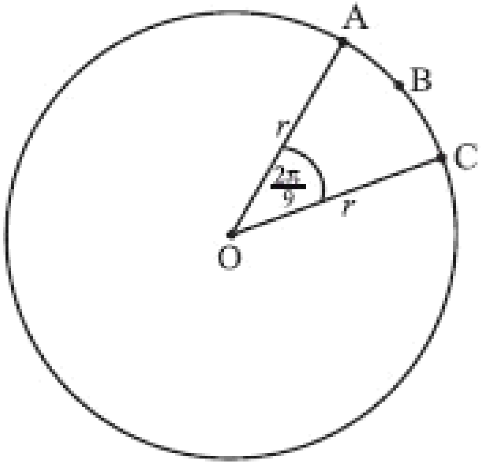
....

....

....

10. [Maximum mark: 6]

The diagram below shows a circle centre O, with radius r.



The length of arc ABC is 3π cm and $\hat{AOC} = \frac{2\pi}{9}$.

(a) Find the value of r. [2]

(b) Find the perimeter of sector OABC. [2]

(c) Find the area of sector OABC. [2]

....

....

....

....

....

....

....

....

....

....

....

....

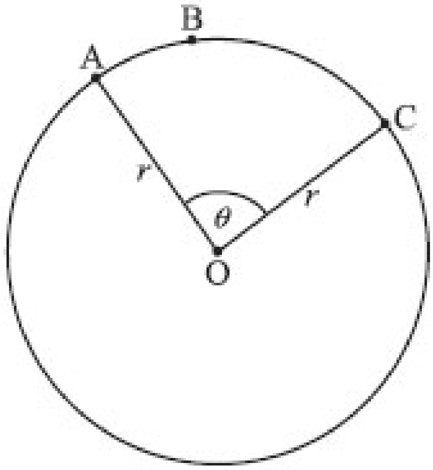
....

....

....

11. [Maximum mark: 7]

The following diagram shows a circle with radius r and centre O. The points A, B and C are on the circle and $\mathrm{A\hat{O}C} = \theta$.



The area of sector OABC is $\frac{4}{3}\pi$, the length of arc ABC is $\frac{2}{3}\pi$.

(a) Find the value of r and of θ. [5]

(b) Find the perimeter of the sector OABC. [2]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

12. [Maximum mark: 9]

The diagram below shows a sector AOB of a circle of radius 15 cm and centre O. The angle θ at the centre of the circle is 2 radians.

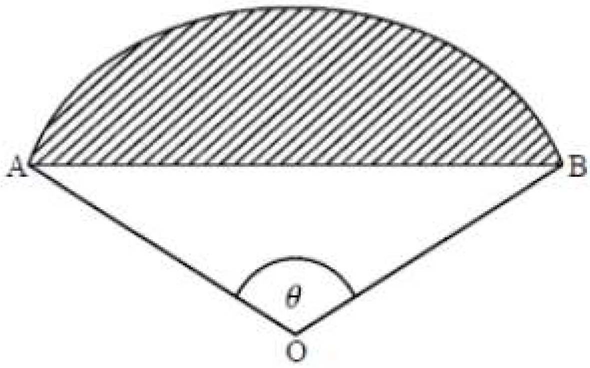


Diagram not to scale

(a) Calculate the area of the sector AOB. [2]

(b) Calculate the area of the shaded region. [3]

(c) Calculate the perimeter of the shaded region. [4]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

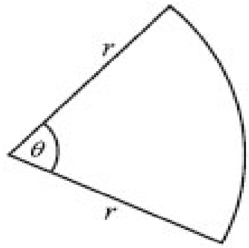
....

....

....

13. [Maximum mark: 5]

The following diagram shows a sector of a circle of radius *r* cm, and angle θ at the centre. The perimeter of the sector is 20 cm.



(a) Show that $\theta = \frac{20-2r}{r}$. [2]

(b) The area of the sector is 25 cm^2. Find the value of *r*. [3]

....

....

....

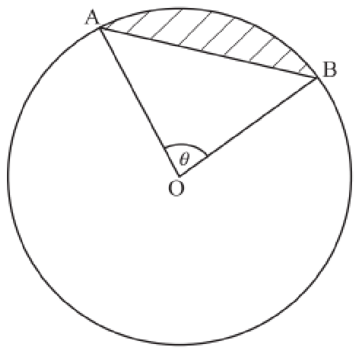
....

....

....

14. [Maximum mark: 5]

The diagram shows a circle centre O and radius *r* cm,, with $\mathrm{A\hat{O}B} = \theta$, $\theta \neq 0$. The area of Δ AOB is three times the shaded area. Find the value of θ.



....

....

....

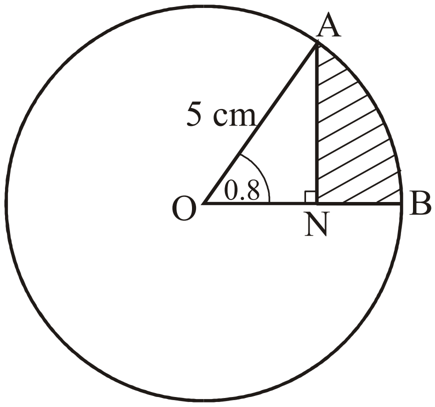
....

....

....

15. [Maximum mark: 6]

The diagram below shows a circle of radius 5 cm with centre O. Points A, B are on the circle, and $A\hat{O}B$ is 0.8 radians. N is on [OB] such that [AN] is perpendicular to [OB].



Find the area of the shaded region.

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

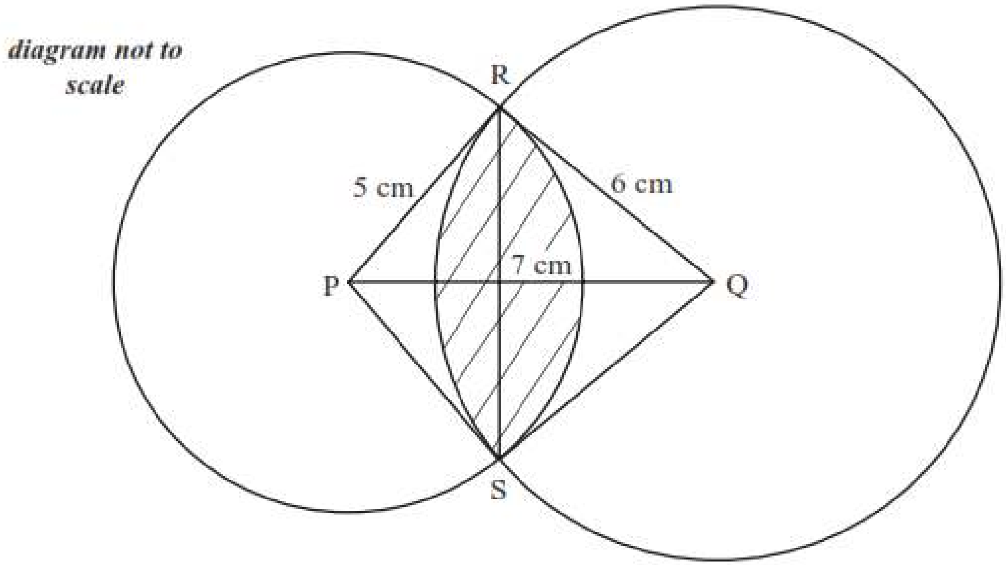
....

....

....

16. [Maximum mark: 6]

The diagram below shows a pair of intersecting circles with centres at P and Q with radii of 5cm and 6cm respectively. RS is the common chord of both circles, PQ is 7 cm.



Find the area of the shaded region.

....

....

....

....

....

....

....

....

....

....

....

....

....

....

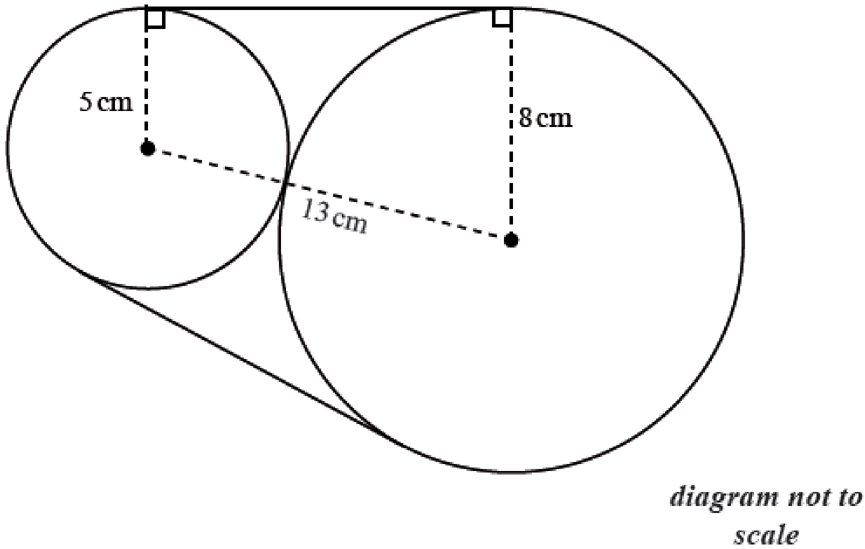
....

....

....

17. [Maximum mark: 6]

Two discs, one of radius 8 cm and one of radius 5 cm, are placed such that they touch each other. A piece of string is wrapped around the discs. This is shown in the diagram below.



Calculate the length of the string needed to go around the discs.

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

TRIANGLES II – AMBIGUOUS CASE

18. [Maximum mark: 8]

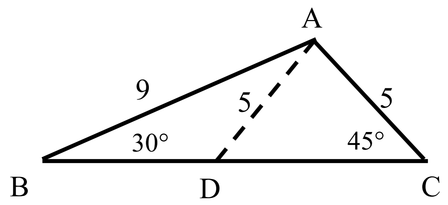
Complete the following table: (angle $\theta \in$ [0° ,180°] to the nearest degree; sin θ to 3sf)

$0° \le \theta \le 90°$	$\sin\theta$
30°	
40°	
50°	0.766
	0.600

$90° \le \theta \le 180°$	$\sin\theta$
150°	
140°	
	0.766
	0.600

19. [Maximum mark: 7]

The diagram shows a triangle ABC with AB=9, AC = AD = 5, $\hat{B} = 30°$, $\hat{C} = 45°$.



(a) Write down the value of the angle $B\hat{A}C$. [1]

(b) Find the value of angle $B\hat{D}A$ and hence the value of angle $B\hat{A}D$. [2]

(c) Find BC. [2]

(d) Find BD. [2]

....

....

....

....

....

....

....

....

....

....

....

20. [Maximum mark: 6]

In the triangle ABC, $\hat{A} = 30°$, $BC = 3$ and $AB = 5$. Find the two possible values of $\hat{B}$.

....

....

....

....

....

....

....

....

....

....

21. [Maximum mark: 7]

In a triangle ABC, $\hat{A} = 35°$, $BC = 4$ cm and $AC = 6.5$ cm. Find the possible values of $\hat{B}$ and the corresponding values of AB.

....

....

....

....

....

....

....

....

....

....

....

....

....

22. [Maximum mark: 6]

In a triangle ABC, $A\hat{B}C = 30°$, $AB = 6cm$, $AC = 3\sqrt{2}$ cm. Find the possible lengths of [BC].

METHOD A: Use Sine rule.

METHOD B: Use Cosine rule.

23. [Maximum mark: 6]

In triangle ABC, $A\hat{B}C = 31°$, $AC = 3cm$, $BC = 5cm$. Calculate the possible lengths of [AB].

24. [Maximum mark: 6]

The diagrams below show two triangles both satisfying the conditions

AB = 20 cm, AC = 17 cm, $A\hat{B}C$ = 50°.

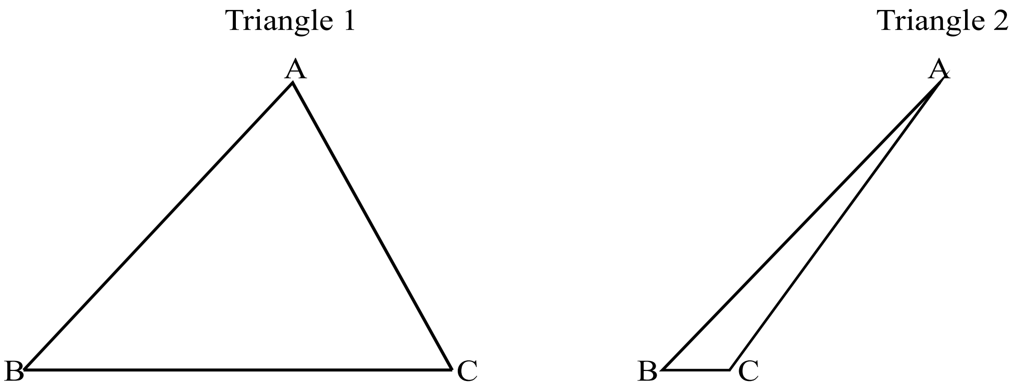


diagram not to scale

Calculate

(a) the size of $A\hat{C}B$ in **Triangle 2.** [3]

(b) the area of **Triangle 1**. [3]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

25. [Maximum mark: 7]

The following diagram shows the triangle ABC.

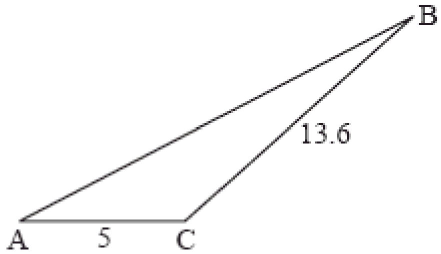


diagram not to scale

The angle at C is **obtuse**, AC = 5 cm, BC = 13.6 cm and the area is 20 cm^2.

(a) Find $A\hat{C}B$. [4]

(b) Find AB. [3]

....

....

....

....

....

....

....

....

....

26. [Maximum mark: 6]

In a triangle ABC, AB = 4 cm, AC = 3 cm and the area of the triangle is 4.5 cm^2.

Find the **two** possible values of the angle $B\hat{A}C$.

....

....

....

....

....

....

....

....

B. Paper 2 questions (LONG)

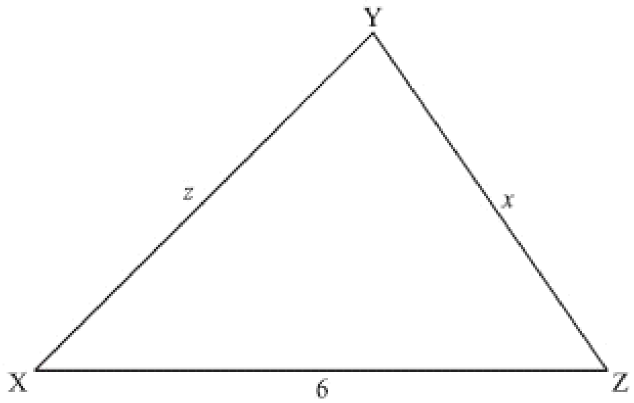
27. [Maximum mark: 20]

(a) Let $y = -16x^2 + 160x - 256$. Given that y has a maximum value, find

(i) the value of x giving the maximum value of y;

(ii) this maximum value of y. [4]

The triangle XYZ has XZ = 6, YZ = x, XY = z as shown below. The perimeter of triangle XYZ is 16.



(b) (i) Express z in terms of x.

(ii) Using the cosine rule, express z^2 in terms of x and $\cos Z$.

(iii) Hence, show that $\cos Z = \dfrac{5x-16}{3x}$. [7]

Let the area of triangle XYZ be A.

(c) Show that $A^2 = 9x^2 \sin^2 Z$. [2]

(d) Hence, show that $A^2 = -16x^2 + 160x - 256$. [4]

(e) (i) Hence, write down the maximum area for triangle XYZ.

(ii) What type of triangle is the triangle with maximum area? [3]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

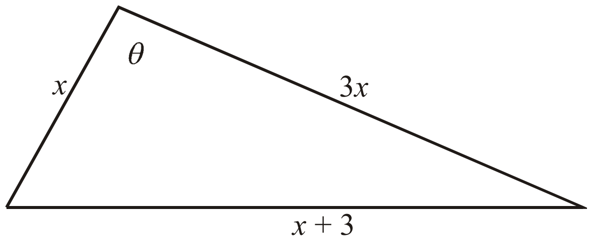
....

....

....

28. [Maximum mark: 18]

The area of the triangle shown below is $2.21\ \text{cm}^2$. The length of the shortest side is x cm and the other two sides are $3x$ cm and $(x + 3)$ cm.



(a) Using the formula for the area of the triangle, express $\sin\theta$ in terms of x. [2]

(b) Using the cosine rule, express $\cos\theta$ in terms of x (simplify your answer). [2]

(c) (i) Using your answers to parts (a) and (b), show that,

$$\left(\frac{3x^2-2x-3}{2x^2}\right)^2 = 1-\left(\frac{4.42}{3x^2}\right)^2$$

(ii) Hence find the possible values of x and the corresponding values of θ, **in radians,** using your answer to part (b) above. [6]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

29. [Maximum mark: 13]

The following diagram shows a circle with centre O and radius 4 cm.

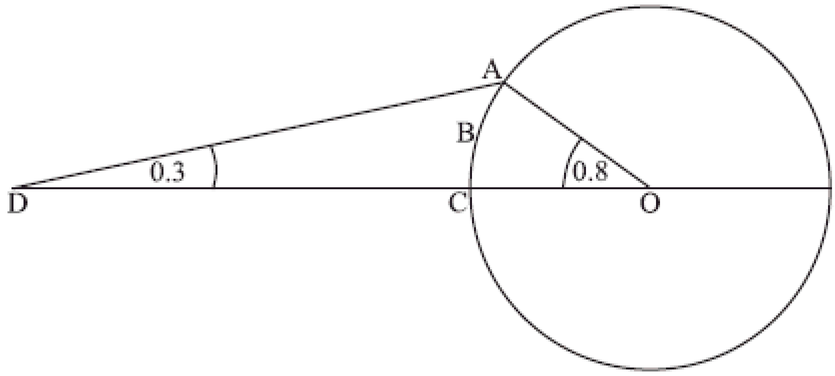


diagram not to scale

The points A, B and C lie on the circle. The point D is outside the circle, on (OC).

Angle ADC = 0.3 radians and angle AOC = 0.8 radians.

(a) Find AD. [3]

(b) Find OD. [4]

(c) Find the area of sector OABC. [2]

(d) Find the area of region ABCD. [4]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

30. [Maximum mark: 14]

The following diagram shows the triangle AOP, where OP = 2cm, AP = 4cm and AO= 3cm

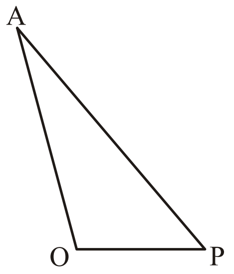


diagram not to scale

(a) Calculate $A\hat{O}P$, giving your answer in radians. [3]

The following diagram shows two circles which intersect at the points A and B. The smaller circle C_1 has centre O and radius 3 cm, the larger circle C_2 has centre P and radius 4 cm, and OP = 2 cm. The point D lies on the circumference of C_1 and E on the circumference of C_2. Triangle AOP is the same as triangle AOP in the diagram above.

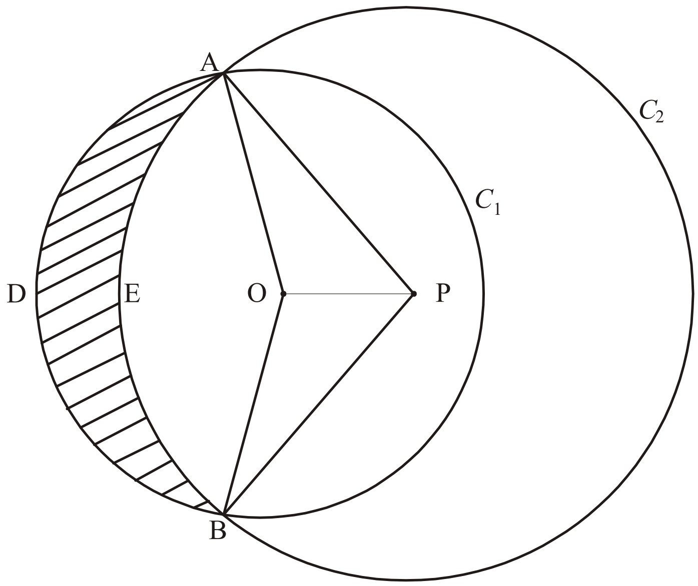


diagram not to scale

(b) Find $A\hat{O}B$, giving your answer in radians. [2]

(c) Given that $A\hat{P}B$ is 1.63 **radians**, calculate the area of

(i) sector PAEB;

(ii) sector OADB. [5]

(d) The area of the quadrilateral AOBP is 5.81 cm^2.

(i) Find the area of AOBE. (ii) Hence find the area of the shaded region AEBD. [4]

....

....

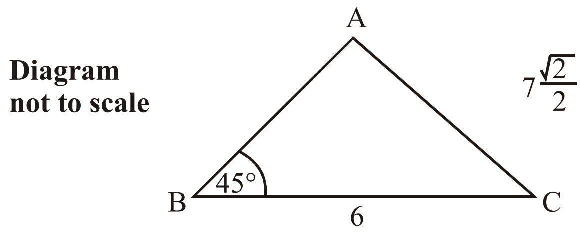
....

....

....
....
....
....
....
....
....
....
....
....
....
....
....
....
....
....
....
....
....
....
....
....
....
....
....
....
....
....
....

31. [Maximum mark: 10]

The diagram shows a triangle ABC in which AC = $7\frac{\sqrt{2}}{2}$, BC = 6, $A\hat{B}C$ = 45°.



(a) Use the fact that sin 45° = $\frac{\sqrt{2}}{2}$ to show that sin $B\hat{A}C$ = $\frac{6}{7}$. [2]

The point D is on (AB), between A and B, such that sin $B\hat{D}C$ = $\frac{6}{7}$.

(b) (i) Write down the value of $B\hat{D}C + B\hat{A}C$.

(ii) Calculate the angle BCD.

(iii) Find the length of [BD]. [6]

(c) Show that $\frac{\text{Area of } \Delta BDC}{\text{Area of } \Delta BAC} = \frac{BD}{BA}$. [2]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....